NON-DIMENSIONAL GROUPS IN THE DESCRIPTION OF FINITE-AMPLITUDE SOUND PROPAGATION THROUGH AEROSOLS

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SUMMARY

Several parameters, which have fairly transparent physical interpretations, appear in the analytic description of finite-amplitude sound propagation through aerosols. Typically, each of these parameters characterizes, in some sense, either the sound or the aerosol. It also turns out that fairly obvious combinations of these parameters yield non-dimensional groups which, in turn, characterize the nature of the acoustic-aerosol interaction. This theme is developed in order to illustrate how a quick examination of such parameters and groups can yield information about the nature of the processes involved, without the necessity of extensive mathematical analysis. This concept is developed primarily from the viewpoint of sound propagation through aerosols, although complimentary acoustic-aerosol interaction phenomena are briefly noted.

NOMENCLATURE

The nomenclature used is consistent with that of reference 1, from which the analytic results discussed in this paper were taken.

C	local wave propagation speed
c_o	infinitesimal sound speed in a clean gas $\equiv (\gamma RT_o)^{\frac{1}{2}}$
c	particulate specific heat
c_p	gas specific heat at constant pressure
H	c/c_v
l	dimensionless length $= \omega x/c_o$
l^*	dimensionless distance to shock formation
l^*_{cg}	dimensionless distance to shock formation in clean gas
m	mass of a single particle
M	equilibrium particulate mass loading $= mn_o/\rho_o$
n	particulate number density
r	radius of a single particle
R	ideal gas constant
T	temperature
U	gas velocity amplitude
u	dimensionless gas velocity
$u^{(i)}$	i^{th} order solution to u
x	dimensional distance
X_o	dimensional piston-displacement amplitude
γ	$= c_p/c_v$
$\delta_T c_o$	= local change in infinitesimal speed of sound due to temperature change
ε	acoustic Mach number $\equiv U/c_o$
μ	gas dynamic viscosity
ρ	gas density
ρ_p	particulate material density

τ particle momentum relaxation time $= (2\rho_p/9\mu)r^2$

ω frequency

INTRODUCTION

Interactions between sound and aerosols have received increasing scientific-engineering attention in recent years. Moreover, many of the more important applications involve high particulate load aerosols and intense acoustic fields. In regard to the latter, the intensities are often sufficiently high that finite-amplitude, or non-linear, effects become important.

Acoustic-aerosol interactions can be examined from two viewpoints. The first results from considering the effect of the aerosol media upon the sound which is propagating through that media. The second results from considering the effect of the acoustic field upon the aerosol itself. Illustrative applications of the former viewpoint include sound propagation through fogs in marine navigation and the attenuation of rocket or jet noise by particulate matter in the exhaust stream. From the latter viewpoint, the most remarkable result is an enhancement of the aerosol agglomeration rate; the result of a marked increase in aerosol particle-particle collision frequency. Perhaps the most promising application of this agglomeration phenomenon is in the conditioning of industrial atmospheric aerosol emissions (ref 2). Very recently, however, interesting prospects for application in the mitigation of LMFBR+ accidents have appeared.

This paper will concentrate on the viewpoint of the influence of the aerosol upon sound propagation. It is this aspect which has succumbed most readily to theoretical treatment and hence presents greater opportunity for examination of meaningful analytical results. It can be our hope, however, that through gaining insight into the important parameters of acoustic-aerosol interactions from this viewpoint, we simultaneously identify those physical parameters of most importantance to phenomena associated with the effect of sound upon an aerosol.

FUNDAMENTAL PARAMETERS

To introduce our approach to the examination of acoustic-aerosol interactions, it is appropriate to first beiefly review what we mean by an aerosol and, secondly, to remind ourselves of the most well-known features of finite-amplitude acoustics. Such a review is worthwhile in itself. But it will also allow us to choose two parameters which we shall use to characterize the aerosol, and two further parameters which we shall use to characterize the acoustic field. Moreover, as we shall see, simple combinations of these parameters can then subsequently be used to allow physical interpretation of the acoustic-aerosol interactions.

+Liquid metal fast breeder reactor.

Of course to properly describe an aerosol or finite-amplitude acoustic field, many more than two parameters for each would be necessary. However, it is the theme of this paper that by choosing what might be considered the two "most important" parameters in each case, a non-rigorous, but interpretively useful, appreciation of the major processes can be gained.

Aerosol

An aerosol may be defined as a suspension of solid and/or liquid particulate matter in a gaseous media. Well-known examples include smokes, mists, fumes and atmospheric dust clouds. The author's preference is to consider an aerosol to be the combined particulate cloud and gaseous bath gas, rather than the particulate cloud itself. We shall use this interpretation through the remainder of this discussion.

The particulate component is typically characterized by classifications such as volatile or non-volatile, spherical or non-spherical particles, monodisperse or polidisperse particulate, size distributions, etc. A monodisperse aerosol (sometimes also referred to as homogeneous) is one which contains particles of only one size (strictly speaking, in only one small size range). Next, in even the briefest outline of the nature of an aerosol, the remarkable phenomenon of continuous and spontaneous particulate agglomeration must be noted. The rate of agglomeration is proportional to the number density squared such that,

$$-\dot{n} \propto n^2. \tag{1}$$

Of course, the appropriate constant of proportionality depends upon several factors, such as, whether the aerosol is quiescent or in turbulent motion, electrical field and charge effects, particulate characterization and, what is of particular import to the phenomena we are treating, the absence or presence of acoustic fields and their nature. Figure 1, which has been abstracted from reference 3, is included to give some feel for the order of magnitudes involved in aerosol dynamics.

To facilitate selection of these parameters which we shall use to characterize the aerosol, it is expedient to choose a simply defined aerosol in order to focus on the major features of the acoustic-aerosol interaction. As such, we shall consider a monodisperse particulate cloud of spherical non-volatile particles, spacially uniformly dispersed throughout an inert, quiescent bath gas which will exhibit no molecular relaxation processes when under the influence of these acoustic fields we shall consider.

Clearly, a parameter of importance will be one which will characterize the size of individual particles, and do so in terms of the gas in which they are immersed, since any parameter chosen to characterize the aerosol must include features of both the particulate cloud and the carrier gas. Such a parameter is the momentum relaxation time, τ, which under assumptions consistent with the application of Stokes Drag Law, becomes

$$\tau = (2\rho_p/9\mu)r^2. \tag{2}$$

The obvious choice for the second aerosol parameter will be one which gives a measure of "how much" particulate matter is present. Again, the "how much" must be given in terms of the bath gas. As such, the natural choice is the mass loading ratio, M, given by,

$$M = (mn)/\rho. \tag{3}$$

We now have two very simple parameters which we shall use to characterize the aerosol. It is of interest to note that one of these, M, can be considered a "how much?" parameter, while the other, τ, is in some sense a "what type?" parameter. It will be useful to retain these simple "how much?" and "what type?" concepts when choosing the two parameters which shall represent the acoustic field.

Finite-Amplitude Sound

Consistent with our approach when we chose a simple aerosol as a vehicle to introduce the aerosol parameters, we shall now direct attention to a simple finite-amplitude acoustic field. In particular, we shall consider a plain progressive wavetrain generated by the sinusoidal motion of a piston at $x = 0$ and propagating into the semi-infinite region $x > 0$. By referring to figure 2, we can review the most well-known phenomenon associated with finite-amplitude sound propagation; that of the distortion of the initial sinusoidal wavetrain into a wavetrain more sawtooth in form containing higher harmonics.

There are two dominant mechanisms which cause this distortion and they do so through their effect on the local wave propagation velocity, C, at each point in the waveform. If we consider C at each point in the wave, we note that it is made up of the linear superposition of three velocities.

$$C = c_o + S_T c_o + u \tag{4}$$

Here, c_o is the quiescent speed of sound, that is the speed of infinitesimal-amplitude sound through the quiescent media. The second term accounts for changes in the local speed of sound due to variations in the temperature of the media caused by the presence of the acoustic field itself. The third term, u, is a convective term, resulting from the fact that the media itself is moving with a local velocity.

If we apply these physical considerations to determine the local speed of sound, C, at each of three points, x_1, x_2 and x_3, in the "early wave form" of figure 2, the mechanism of wave form distortion is easily understood. At x_3 compression effects have increased the temperature of that part of the wave above that in the quiescent gas. As such, $\delta_T C_o$, is positive. Moreover the convective velocity u is positive. Thus, C at x_3 is greater than c_o, and that "part" of the wave moves faster in the direction of propagation than would an infinitesimal amplitude wave. By similar arguments it is apparent that C at x_2 equals c_o and C at x_1 is less than c_o. These combined effects lead to the distortion shown by the "later wave form" of figure 2.

Now the important thing from our point of view is that both $\delta_T c_o$ and u are a *direct consequence* of the finite-amplitude nature of the acoustic field. As

such, in seeking parameters to characterize the acoustic field, we necessarily require a parameter which will measure the magnitude of finite-amplitude effects, or, the degree to which the field is a "finite-amplitude" field. The most appropriate parameter for this purpose is the acoustic Mach number, ε, given by,

$$\varepsilon = U/c_o. \tag{5}$$

Note that U is the acoustic velocity amplitude and not the local convective velocity, u, discussed earlier.

Now by comparison with the parameters introduced to describe the aerosol, it is evident that ε may be thought of as a "how much?" parameter. And this leaves us with the choice of a "what type?" parameter, which for the acoustic field is clearly ω, the fundamental sinusoidal frequency. Interestingly, in terms of the displacement amplitude, X_o, and for sinusoidal motion, the two acoustic parameters are related through the expression,

$$\varepsilon = (wX_o)/c_o. \tag{6}$$

Towards the end of this "early evolution" stage where non-linear effects bring about a relatively rapid transfer of energy from the fundamental to the higher harmonics, the waveform can resemble a series of low amplitude shocks. This distance to shock-formation has been given by Blackstock (ref 4) as,

$$l^*_{cg} = 2/(\gamma + 1)\varepsilon. \tag{7}$$

At shock-formation, the ratio of magnitudes of the fundamental to its harmonics are "semi-stable", resulting from a balance between the concomitant processes of energy flow from the lower harmonics to the higher harmonics, and the proportionally greater dissipation of the higher harmonics. This concept was first introduced by Fay (ref 5). The development of the wavetrain beyond the point of shock-formation to extinction might be thought of as the "late evolution" phase during which the progressive dissipation of the energy associated with the waveform causes the "semi-stable" waveform to decay to an infinitesimal sinusoidal form.

Before leaving the interpretation of these four parameters, it is appropriate to give some feeling for typical orders of magnitudes involved in units associated with either aerosol science or non-linear acoustics. This is presented in Table 1.

M	(grains/ft³)	(gms/m³)	ε	SPL(dB)	(Watts/m²)	τ(sec)	r(μm)
10^{-4}	0.057	0.013	10^{-4}	114	$2.5\text{x}10^{-1}$	$1.2\text{x}10^{-7}$	0.1
10^{-3}	0.57	0.13	10^{-3}	134	2.5x10	$1.2\text{x}10^{-5}$	1
10^{-2}	5.7	1.3	10^{-2}	154	$2.5\text{x}10^{3}$	$1.2\text{x}10^{-3}$	10
10^{-1}	57	13	10^{-1}	174	$2.5\text{x}10^{5}$	$1.2\text{x}10^{-1}$	100

TABLE 1: Illustrative conversions of parameters M, ε and τ, for $\rho_p = 1$ gm/cm³, $\rho_o = 1.29\text{x}10^{-3}$ gm/cm³, $\mu = 1.83\text{x}10^{-4}$ poise.

Intuitive

Of the four parameters, the two "what type?" parameters, $\tau[t]$ and $\omega[t^{-1}]$, have inverse units, and their simple product, $\omega\tau$, is a non-dimensional group which represents the physical ratio:

$$\omega\tau = \frac{\text{time of particle dynamic relaxation}}{\text{time of acoustic cycle}} . \tag{8}$$

As such, we expect the magnitude of $\omega\tau$ will tell us something about how effectively the aerosol and sound are coupled. In particular, as $\omega\tau \to 0$, the time of particle dynamic relaxation is very short with respect to the time of an acoustic cycle, and hence we expect the particles to behave much as if they were an element of fluid in the bath gas. That is, we expect their presence to play a minimal role affecting the sound being transmitted through the aerosol media. As $\omega \to 1$ the dynamic relaxation time of the aerosol particles is of the same order as the time of an acoustic cycle, and hence we might expect, in some sense, a maximum acoustic-aerosol interaction. As $\omega\tau \to \infty$, the long dynamic relaxation time with respect to an acoustic cycle indicates that the aerosol particles are essentially stationary. We therefore expect that the presence of the acoustic field has minimal effect upon the aerosol. We might further expect that although the aerosol could influence the sound, it would do so in only a minor way, since the sound can be expected to propagate primarily through the gaseous media in the interstices between particles, which is large w.r.t. the particle volume.

The two remaining "how much?" parameters combine to give a non-dimensional group, M/ε, even more simply interpreted. Specifically,

$$M/\varepsilon = \frac{\text{"how much?" particulate matter}}{\text{"how much?" sound}}. \tag{9}$$

With this interpretation, we expect that as $M/\varepsilon \to 0$, finite-amplitude sound effects will dominate processes of interest. As $M/\varepsilon \to 1$, we expect that finite-amplitude and aerosol effects influence various phenomena with approximately equal importance. As $M/\varepsilon \to \infty$, we expect that the presence of the aerosol, rather than that of finite-amplitude nature of the acoustic field, will be of predominant importance. Of course these M/ε interpretations should be viewed in terms of the magnitude of the associated $\omega\tau$ parameter which gives information on the effectiveness of the acoustic-aerosol interaction. That is, if the $\omega\tau$ product indicates a weak acoustic-aerosol coupling, the significance of the M/ε parameter might be unimportant á priori.

Analytic

We now consider the role of the preceeding acoustic-aerosol interaction parameters by examining the analytic results of reference 1. In particular, we shall examine the influence of the aerosol upon waveform distortion.

The assumptions and analytic details of the results we shall consider are presented in reference 1, and will not be repeated here. The work involved a perturbation solution of a set of equations and boundary conditions which described the attenuation, dispersion and harmonic growth, of an initially sinusoidal finite-amplitude plain progressive wavetrain propagating towards infinity. The solution, in terms of the dimensionless gas velocity, u, was given in the form:

$$u = \varepsilon u^{(1)} + \varepsilon^2 u^{(2)} + \varepsilon^3 u^{(3)} + \ldots \tag{10}$$

Although not rigorously correct (see ref. 1), it is possible for the purposes of our discussion to consider each term in this expansion, $\varepsilon^i u^{(i)}$, to be the i^{th} harmonic. With this interpretation we consider the magnitude of waveform distortion by examining the evolution of $u^{(2)}$ with increasing distance from the initial sinusoidal motion.

First consider the case, shown in figure 3, where the $\omega\tau$ product is held constant at unity and M is increased from 0 to 10^{-1}. The M = 0 result corresponds to the growth of the second harmonic in a clean gas. As we increase the amount of particulate matter, the harmonic growth is retarded as the energy is removed from the harmonic by particulate-gas dissipative mechanisms.

Turning to the influence of the parameter $\omega\tau$, we can consider the case for which we hold the mass loading ratio constant at $M = 10^{-2}$. This is illustrated in figure 4 which shows the influence of the $\omega\tau$ parameter over the range $0 \leqq \omega\tau \leqq \infty$. In spite of the fact that the particulate loading is non-negligible, at $\omega\tau = 0$ the presence of the particulate matter does not alter the growth of the second harmonic from that which it would be in a clean gas. As the $\omega\tau$ product moves to $\omega\tau = 10^{-1}$, the acoustic-aerosol coupling improves, and the growth of the second harmonic is somewhat retarded over that which would be found in a clean gas. If we further increased $\omega\tau$ through unity to infinity, the maximum retardation of harmonic growth occurs at approximately $\omega\tau = 1$, after which the aerosol influence diminishes until, as $\omega\tau \to \infty$, the sound propagates as if there were no particulate matter present.

We see how the presence of particulate matter, as given by the mass loading ratio M, acts to *retard* the rate at which the growth of the second harmonic distorts the original sinusoidal waveform. But the effectiveness by which the aerosol retards the distortion is strongly affected by the effectiveness of the acoustic-aerosol coupling as indicated by the $\omega\tau$ parameter.

The next question which can be examined from this parametric approach is that of the distance to shock-formation. Or, what may be considered the same thing, the demarkation between the "early evolution" and "late evolution" stages of waveform development.

Analytically, this problem has been approached by comparing the magnitude of terms in the perturbation solution of equation (10). In reference 1 it is shown that, in an inviscid clean gas, the ratios of the second order solution to the first, and the third to the first, at the point of shock-formation, become,

$$\left|\varepsilon u^{(2)}/u^{(1)}\right| = \tfrac{1}{2} \tag{11}$$

and

$$|\varepsilon^2 u^{(3)}/u^{(1)}| = 3/8 \tag{12}$$

respectively. Either equation (11) or (12) may be used as a shock-formation criterion for the case $M = 0$. If, however, it is assumed that the relative amplitudes of the harmonics at the point of shock-formation in an aerosol are also given by equations (11) and (12), the difference between the two predictions will give some indication of the uncertainty associated with this assumption. This was the approach taken in reference 1 and the results are shown in figure 5.

We note that the distance to shock-formation in an aerosol is always greater than the distance to shock-formation in the clean gas. Thus, that which was indicated by figures 3 and 4 - that the presence of an aerosol retards the rate at which an initially monochromatic finite-amplitude wave distorts - is reinforced by the shock-formation distance results. But the matter of particular interest which is illustrated by figure 5, is the role played by the interaction parameters M/ε and $\omega\tau$.

If we direct attention to the case $\omega\tau = 1$, we note that at $M/\varepsilon \approx 1$ the distance to shock-formation in the aerosol is about 1.35 times that found in a clean gas. On the other hand, as M/ε decreases from unity the distance to shock-formation approaches that found in the clean gas, while as M/ε becomes greater than unity, the shock-formation distance rapidly approaches infinity and the system moves into a regime where shock-formation is precluded. Thus our earlier speculation, that the M/ε interaction parameter should be a measure of the relative importance of aerosol effects as compared to finite-amplitude effects is realized. Put simply, for the case $M/\varepsilon = 0.1$, the wave distortion proceeds essentially as if there were no aerosol present and is determined totally by the magnitude of the acoustic field. Conversely, for the case $M/\varepsilon \approx 10$, the acoustic field is damped out so rapidly by the presence of the aerosol that finite-amplitude distortion effects have no opportunity to significantly alter the harmonic ratios. In both cases, this is independent of the *absolute* magnitude of either M or ε.

Now consider the effect of the parameter $\omega\tau$, by examining the cases $\omega\tau = 0.1$ and $\omega\tau = 10$. As we move away from $\omega\tau = 1$, the coupling between the acoustic field and the aerosol becomes less effective, thereby weakening the influence of the aerosol upon waveform evolution, and allowing this evolution to proceed more closely to that which it would in a clean gas. Again, this conforms not only to the results of figures 3 and 4, but also to our intuitive expectations discussed earlier.

Before leaving figure 5, it should be noted that the two harmonic ratio criteria do predict somewhat different shock-formation distances. This discrepancy becomes marked as the ratio $l^*/l^*_{cg} \simeq 2$ or greater, and results from the fact that the semi-stable waveform characteristic of the newly formed shock can be quite different in an aerosol than in a clean gas. This, and other matters, will be mentioned briefly in the next section.

MORE GENERAL THEORIES AND ADDITIONAL PARAMETERS

The theme of this paper has been to introduce a means of performing order-of-magnitude evaluations of the relative importance of different processes in finite-amplitude sound propagation through aerosols, by means of quick examination of the four parameters, ω, τ, M and ε, as well as their combination in the interaction parameters M/ε and $\omega\tau$. Nevertheless, the subject should not be left without noting that additional parameters enter more general theories, and giving some indication of where these theories may be found in the literature.

The $\omega\tau$ product has been found in acoustic-aerosol literature for a number of years, and is perhaps best known from the monograph by Mednikov (ref 6) in which the motion of individual particles is given in terms of a *specified* acoustic field. This same parameter plays a major role in the formulation by Temkin and Dobbins (ref 7) of the attenuation and dispersion of infinitesimal-amplitude sound propagation through aerosols. In the event other transport phenomena play a role, such as heat or mass transfer, other "what type?" products, $\omega\tau_i$, where i corresponds to the transport process of interest, can become important. Sometimes, such as in the case of the theory drawn upon for the present paper, even though one of these transport processes cannot be neglected (heat transfer in the present case), the judicious choice of other parameters (in our case setting $H = 1$) can remove this additional dependence such that the problem collapses to a dependence on $\omega\tau$ only. In other cases, such as when volatile aerosols (ref 8) or compressibility of the particles (ref 9) are treated, it may be somewhat more difficult to collapse all "what type?" parameters into one.

The treatment of polydisperse aerosols is fairly readily accomplished by treatments analogous to those referred to here, by the introduction of integrals over the size distribtution using the appropriate size dependent parameter, such as τ, as a weighting function (e.g., references 1, 7, 10). If the semi-stable waveforms of the "late evolution" regime are of interest, in addition to the parameters we have introduced in the body of this paper, the parameter H plays a major role and cannot be neglected. This "late evolution" regime is treated by Davidson (ref 11).

CONCLUSIONS

It is the author's view that if an estimate of the relative magnitude, or importance, of different effects encountered in phenomena associated with finite-amplitude sound propagation through aerosols are of interest, a fairly quick appraisal of these can be made, by examining the magnitude of what are probably the two most important acoustic-aerosol interaction parameters M/ε and $\omega\tau$. In many cases, examination of these parameters will show that one or more effects will either dominate, or are unimportant, to the situation being considered. Moreover, it may be hoped that by considering these groups (which arise naturally in rigorous theoretical treatments) from the physical viewpoints discussed, the underlying physical mechanisms may be better appreciated.

REFERENCES

1. Davidson, G.A.; and Scott, D.S.: Finite-Amplitude Acoustics of Aerosols, J. Acoust. Soc. Amer., vol. 53, no. 6, June 1973, pp. 1717-1729.
2. Scott, D.S.: A New Approach to the Acoustic Conditioning of Industrial Aerosol Emissions, J. Sound Vib., vol. 43, no. 4, Dec. 1975, pp. 607-619.
3. Stanford Research Institute Journal, Third Quarter, 1961. (full chart reprints available from Dept. 300).
4. Blackstock, D.T.: J. Acoustic Soc. Amer., vol. 34, 1962, p. 9.
5. Fay, R.D.: J. Acoustic Soc. Amer., vol. 3, 1931, p. 222.
6. Mednikov, E.P.: Acoustic Coagulation and Precipitation of Aerosols, English translation; Consultants Bureau, N.Y., 1965.
7. Temkin, S.; and Dobbins, R.A.: Measurements of Attenuation and Dispersion of Sound by an Aerosol, J. Acoust. Soc. Amer., vol. 40, 1966, pp. 1016-24.
8. Cole, J.E.; and Dobbins, R.A.: Measurements of the Attenuation of Sound by a Warm Fog, J. Atmos. Sc., vol. 21, 1971, pp. 202-209.
9. Morfey, C.L.: Sound Attenuation by Small Particles in a Fluid, J. Sound Vib., vol. 8, 1968, pp. 156-170.
10. Davidson, G.A.; and Scott, D.S.: Finite-Amplitude Acoustic Phenomena in Aerosols from a Single Governing Equation, J. Acoust. Soc. Amer., vol. 54, no. 5, Nov. 1973, pp. 1331-1342.
11. Davidson, G.A.: A Burgers' Equation Approach to Finite-Amplitude Acoustics in Aerosol Media, J. Sound Vib., vol. 38, 1975, pp. 475-495.

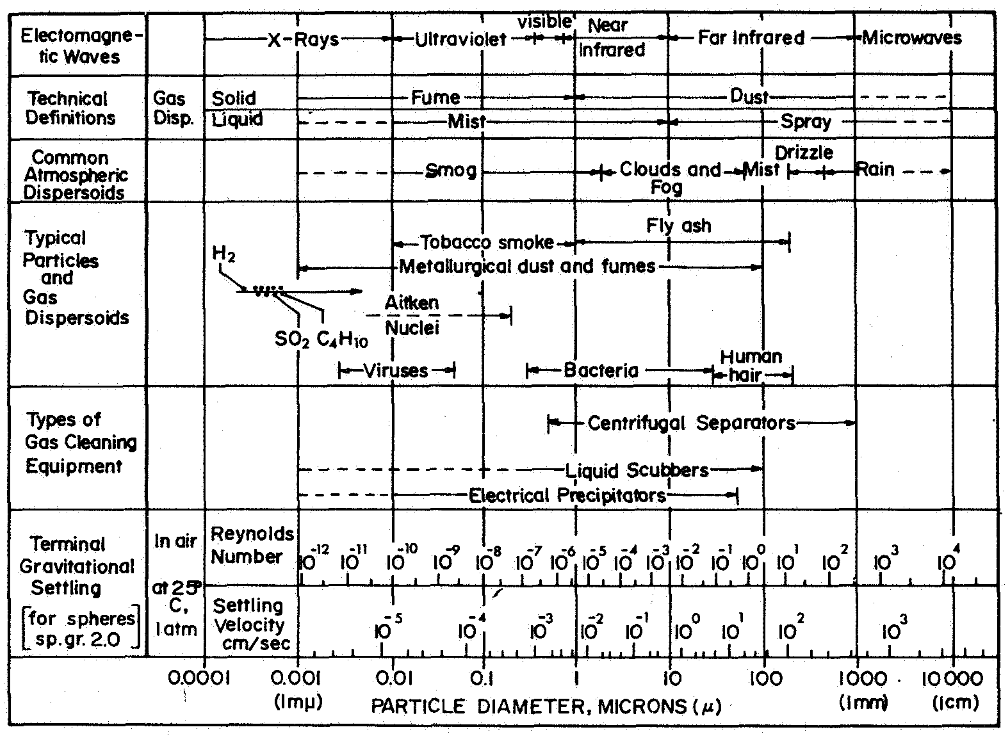

Figure 1.- Characterization of some aerosols.

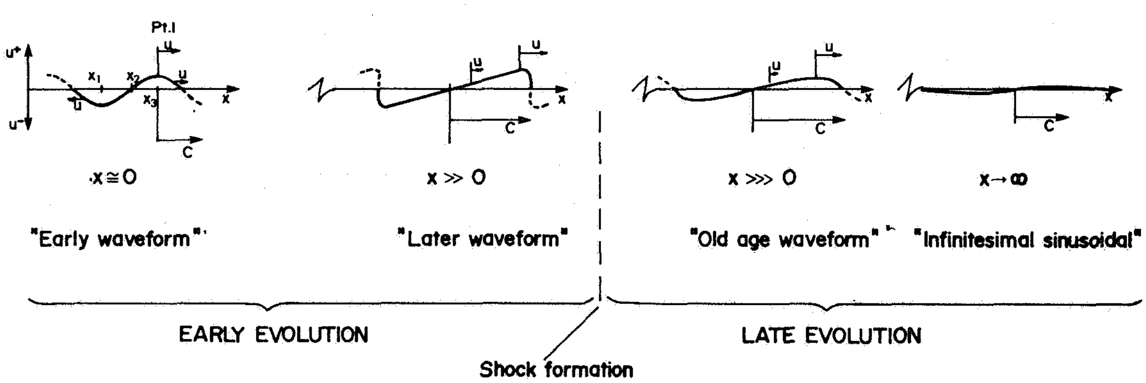

Figure 2.- Life-cycle of finite-amplitude progressive waves.

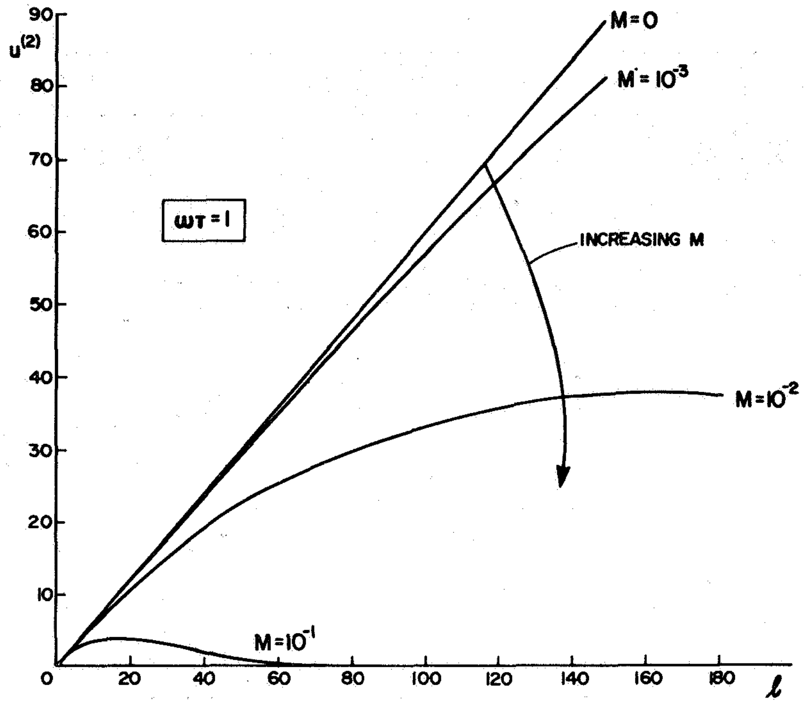

Figure 3.- Magnitude of $u^{(2)}$ versus dimensionless distance from wavetrain berth as finite-amplitude pure sinusoid showing functional dependence on M.

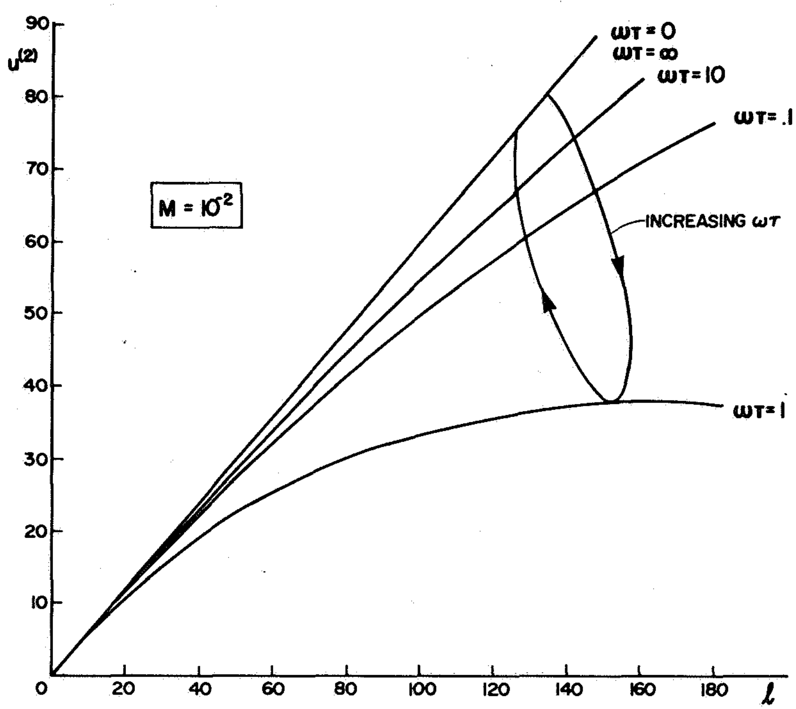

Figure 4.- Magnitude of $u^{(2)}$ versus dimensionless distance from wavetrain berth as finite-amplitude pure sinusoid showing functional dependence on $\omega\tau$.

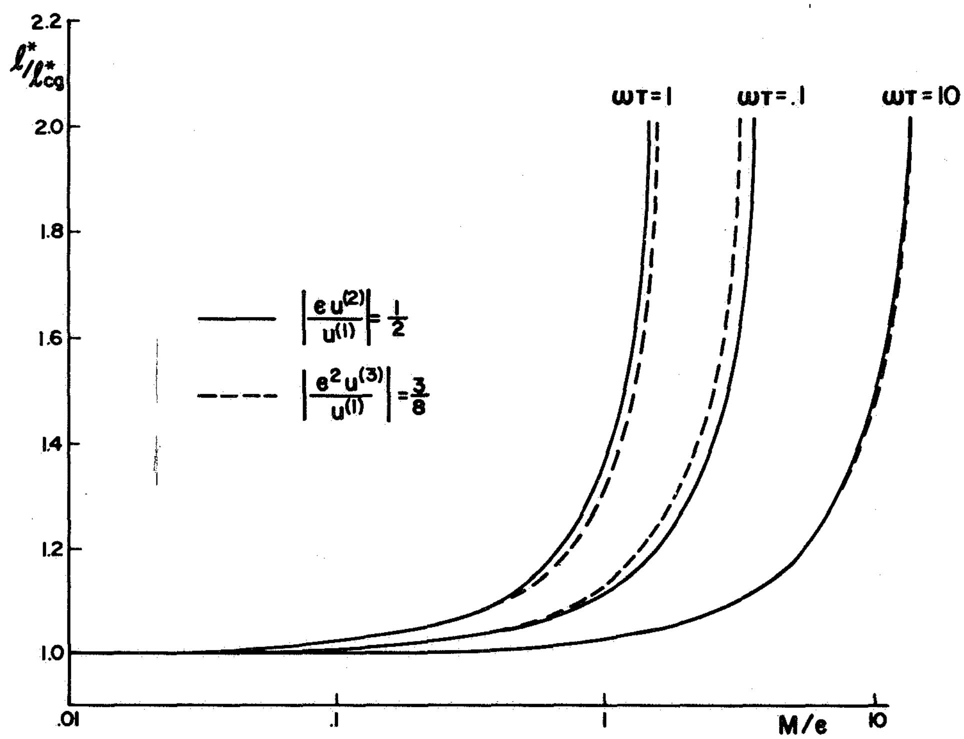

Figure 5.- Normalized dimensionless shock-formation distance versus M/ε parameter as a function of ωτ.